New Mechanism of Quantum Oscillations in the Superconducting Mixed State ($H_{c1} \ll B \ll H_{c2}$)

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We argue that inhomogeneity inherent to the presence of periodic supercurrents in the vortex lattice sorts excitations by energies into the ones that are spatially localized and those that perform motion along large Larmour orbits. This energy threshold results in a new mechanism for the de Haas-van Alphen oscillations which enhances oscillations at $B \ll H_{c2}$, even for an isotropic superconductor with a constant gap. We suggest that the mechanism is of a general character and can cause the slow decay of the de Haas-van Alphen effect when the field, B, decreases below H_{c2}.

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The de Haas-van Alphen (dHvA) effect in the superconducting (SC)-state was recently reported at magnetic fields, B, surprisingly smaller than the upper critical field value, H_{c2} [1,2]. Theoretically, the dHvA-signal was expected to decay faster (see discussion and references in [3]). In the experiments [1,2] the effect remains observable at B as low as $0.3 \div 0.4 H_{c2}$, where the average SC-gap is thought to be too large for an electron to perform a circular motion along orbits with the large Larmour radius, $r_L \sim v_F/\omega_c$.

Inhomogeneity of the current distribution and of the SC-order parameter itself cause serious difficulties for a rigorous theoretical treatment. Therefore in [3] an extreme limit of the lattice of isolated vortices,

$$\xi_0 \ll d \ll \delta_L \ ; \quad H_{c1} \ll B \ll H_{c2} \tag{1}$$

has been chosen to find out whether any symmetry zeroes in the gap function could enhance the dHvA effect in the SC-state (in (1) ξ_0 is the coherence length, d- the vortex lattice period, δ_L- the penetration depth).

The results [3] confirmed expectations [4] that in the 3D-case existence of a symmetry line of zeroes in the gap may restore the dHvA effect for some field directions, although the effect is much weaker than predicted in [4] due to scattering of electrons on flux lines.

As for a "d-wave" gap in layered superconductors, the levels' systematics is such [3] that it forbids their crossing the chemical potential with the field change – the mechanism known to be the essence of the dHvA effect in normal metals. No qualitative difference is expected, hence, between a "d-wave" and an ordinary anisotropic gap as far as the dHvA effect is concerned.

Chances to observe the dHvA oscillations for a superconductor in the regime (1) are rather vague, for the signal rapidly decreases below the experimental resolution. Common predictions for the signal's amplitude would give an exponential factor, $\exp(-\Delta/\omega_c) \ll 1$, where Δ is a gap scale and ω_c the cyclotron frequency (e.g., see [5]).

On the theory part however, the limit (1) suggests significant simplifications, because the spatial distribution of the field, currents, and the gap are all well known from the phenomenological consideration [6]. Main contributions are expected to come from the "bulk" since the volume occupied by vortex cores is small.

In that which follows the field range (1) is chosen again to *demonstrate* the existence of some *new* quantum oscillations mechanism which is specific to the SC-state and is originated by spatial variation of the local density of states. Although we expect that the effect should have a more general character, we have chosen the regime (1) and the isotropic gap for its simplicity.

In the SC-state an excitation bears electron- or hole-like features to the extent its energy, $\varepsilon(\mathbf{p}) = \sqrt{v_F^2(p-p_F)^2 + \Delta^2}$, exceeds Δ. Taking supercurrents into account, the local spectrum becomes

$$E(\mathbf{p}) = \varepsilon(\mathbf{p}) + \mathbf{p}\cdot\mathbf{v}_s(\mathbf{r}) \tag{2}$$

where $\mathbf{v}_s(\mathbf{r})$ stands for distribution of superfluid velocity. Hence, the second term in (2) generates a potential relief, such that an excitation with a momentum $\mathbf{p}$ may, or may not, perform infinite motion depending on whether its energy exceeds, or not, the value $\Delta + max|\mathbf{p}\cdot\mathbf{v}_s(\mathbf{r})|$. An excitation (2) experiences a Lorentz force on the part of the magnetic field. Whether an excitation may perform an itinerant motion is equivalent to the condition that such excitations can encircle a large Larmour orbit. The additional potential in (2), thus, imposes an energy boundary between "localized" and "extended" states. The latter may contribute to magnetization.

To show that the above simple arguments, indeed, lead to a new contribution to quantum oscillations in the SC-state, we explore further the equations derived in [3] for a superconductor in a magnetic field. The method [3] consists of averaging of the Gor'kov equations over classical trajectories in the magnetic field.

The core of the method is given by eqs. (24-27) of [3]. We use the same notations [3] below and start with deriving the new Schrodinger equation with the $h(\varphi)$-terms (eq. (31) [3])included and $\Delta \equiv$const.:

$$-\omega_c^2 y'' - 2E h(\varphi) y = (E^2 - \Delta^2) y \tag{3}$$

Note that the $h(\varphi)$-term of eq. (31) [3] is nothing but the Doppler shift in (2). In the derivation of (3) terms

of the order of $h^2(\varphi)$ and $\omega_c(dh(\varphi)/d\varphi)$ were omitted. To estimate the second term recall that the dependence $h(\varphi)$ on φ originates from the dependence of $\mathbf{v}_s(\mathbf{r}(t))$ on t: $t \to \varphi/\omega_c$. An electron moving along the orbit with a velocity v_F senses changes in $\mathbf{v}_s(\mathbf{r})$ with a frequency v_F/d. A change on distances $\sim d$ is therefore equivalent to its change on the scale, $\delta\varphi$, of the order of

$$\delta\varphi \sim (d\omega_c/v_F) \sim d/r_L \tag{4}$$

The $h(\varphi)$-term being $h(\varphi) \sim v_F/d$, one obtains:

$$h^2(\varphi) \sim \omega_c h'(\varphi) \sim v_F^2/d^2 \sim \Delta^2(\xi_0/d)^2$$

In (3) at $E \sim \Delta$ one has:

$$Eh(\varphi) \sim \Delta^2(\xi_0/d) \ll \Delta^2 \tag{5}$$

As it was argued in [3], the oscillatory part of the magnetization is contained (eq. (6') [3]) in the expression:

$$M = -\frac{\mu e}{2\pi c}\sum_{\lambda,\sigma}\left[\overline{|u_\lambda(\varphi)^2|}n(E_\lambda)\right. \tag{6}$$

where the index λ numerates the eigenvalues. At $T = 0$ $n(E_\lambda) \equiv 1$ (all states below zero are occupied). The bar $\overline{(\ldots)}$ means the normalization integral, $(2\pi)^{-1}\int_0^{2\pi}(\ldots)d\varphi$. Oscillatory effects, if any, come from the vicinity of the chemical potential [3]:

$$\mu = \omega_c N_0 + \bar{\mu} \tag{7}$$

The chemical potential being large, $\mu/\omega_c \gg 1$, the oscillatory part of magnetization in (6) does not depend on a specific N_0. The periodic (in B^{-1}) pattern in M of eq. (6) may, therefore, be expressed as a function of

$$\kappa = \bar{\mu}/\omega_c \tag{8}$$

with κ varying in the interval (0,1). The eigen functions $y(\varphi)$ themselves are not periodic. The periodicity conditions are to be imposed on:

$$y(\varphi)\exp\{-i\kappa\varphi\} \tag{9}$$

With (8,9) in mind, the problem of solving eq.(3) becomes equivalent to the one of finding the band structure of a particle moving in the periodic potential, $2Eh(\varphi)$, where κ (8) plays the role of a quasimomentum, $\omega_c^2 \Rightarrow (2m_{eff})^{-1}$ and $E^2 - \Delta^2$ being the "energy".

Summation over λ in (6) is routinely replaced by the integration by making use of the Poisson formula:

$$\sum_{n=-\infty}^{+\infty}\delta(\lambda - n) = \sum_{K=-\infty}^{+\infty}\exp(2\pi i K\lambda) \tag{10}$$

Applying (10) to eq. (6), integrating over λ by parts, one would arrive at terms of the form:

$$M_{osc} = \frac{2i\mu e}{(2\pi)^2 c}\sum_K \frac{1}{K}\int_{-\infty}^{+\infty} e^{2i\pi K\lambda}\frac{d}{d\lambda}\left(\overline{|u_\lambda(\varphi)|^2}\right)d\lambda \tag{11}$$

first noticed in [3]. Eqs. (10) and (11) have no value unless a relation between λ and energy is established. Below we will construct a function

$$\lambda(E) = \Phi(E)/2\pi \tag{12}$$

such that all the eigenvalues in (6) are determined as in (10), by the provision:

$$\lambda(E_\lambda) = n \tag{13}$$

If $\Phi(E)$ were known on the real E-axis, it then may be analytically continued onto the complex plane.

With this in mind first simplify notation in (3)

$$-\varepsilon = E + \Delta - h_{max}(\varphi_o) \tag{14}$$

$h_{max}(\varphi_0)$ is the maximum of $h(\varphi)$ along a given trajectory. One may assume $\varphi_0 = 0$. Eq. (3) becomes:

$$-\omega_c^2 y'' + 2\Delta(h(\varphi) - h_{max})y = 2(-\varepsilon)\Delta y \tag{3'}$$

Even though $h \sim v_F/d \sim \Delta(\xi_0/d) \ll \Delta$, the ratio

$$\Delta h/\omega_c^2 \gg 1 \tag{15}$$

is large. In the WKB-approach:

$$y(\varphi) = ay_+(\varphi) + by_-(\varphi) \tag{16}$$

where $y_\pm(\varphi)$ are of the form [7]:

$$y_\pm(\varphi) = A(S')^{-1/4}\exp\left(\pm i\int_0^\varphi S'(\varphi)\right) \tag{16'}$$

Here A is the normalization factor and

$$S'(\varphi) = \left(\sqrt{2\Delta}/\omega_c\right)\sqrt{h_{max} - h(\varphi) - \varepsilon} \tag{17}$$

At $(-\varepsilon) > 0$ the quantization condition (taking (9) into account) gives:

$$S(2\pi, -\varepsilon_n) = \frac{\sqrt{2\Delta}}{\omega_c}\int_0^{2\pi}\sqrt{h_{max} - h(\varphi) - \varepsilon_n} = 2\pi n + 2\pi\kappa \tag{18}$$

Eq. (18) at $|\varepsilon| \gg \Delta$ matches the spectrum of free electrons in the magnetic field (in presence of the flux currents). At $(-\varepsilon) < 0$ one would obtain "localized" states. Let us justify first this last statement.

The "potential," $h(\varphi) - h_{max} < 0$ in (17) is rather irregular and has many minima alternating with local maxima on distances of order of $\delta\varphi$, (4). The probability to tunnel between minima is measured by the value of $\exp(-|\delta S|)$ with $\delta S \sim (d/\xi_0)^{1/2} \gg 1$, i.e. is small enough to consider barriers as impenetrable ones.

The two WKB-branches, one at $(-\varepsilon) > 0$ and the other for $(-\varepsilon) < 0$, cannot together form a function $\Phi(E)$ such as eq. (12). The WKB-approach is invalid at $(-\varepsilon)$ too close to the maximum of $h(\varphi)$. Indeed, expanding (18) at small $(-\varepsilon) > 0$ results in the singularity:

$$S(2\pi, -\varepsilon) \simeq S(2\pi, 0) + \frac{(-\varepsilon)}{2\omega_c a^{1/2}} \left[\ln \left(\frac{\Delta a(\delta\varphi)^2}{-\varepsilon} \right) \right] \quad (19)$$

For $h(\varphi)$ close to the maximum we have chosen:

$$h(\varphi) = h_{max} - (a\Delta\varphi^2/2) \quad (20)$$

with $a\Delta \sim v_F/d(\delta\varphi)^2$.

Thus, in the WKB-approximation there are two sorts of solutions: those of the form of eq. (16) at $(-\varepsilon) > 0$, and the "localized" states at $(-\varepsilon) < 0$. It is useful to show how these findings are connected with our intuitive expectations regarding the role of the Doppler term (2).

The solutions $u_\lambda(\varphi)$ and $v_\lambda(\varphi)$ must be normalized together: $\overline{|u_\lambda(\varphi)|^2} + \overline{|v_\lambda(\varphi)|^2} = 1$. There are two useful auxiliary relations:

$$\overline{|u^2|} = (1/2) \left\{ \overline{|y|^2} + (i\omega_c/2E)\overline{(y^* y' - y y^{*\prime})} \right\}$$
$$\overline{|v^2|} = (1/2) \left\{ \overline{|y|^2} - (i\omega_c/2E)\overline{(y^* y' - y y^{*\prime})} \right\} \quad (21)$$

which may be derived with the same accuracy as (3). One concludes from (21) that $\overline{|y|^2} = 1$, and $\overline{|u_\lambda|^2} = \frac{1}{2}$ *independently on the energy for all localized states* for which the wave functions are real. Therefore there is, indeed, a threshold singularity in M_{osc} (11)– "localized" states give no contribution into M_{osc}. To study the phenomenon rigorously one must go beyond the WKB-accuracy.

At the energy close to the maximum of the potential in eq. (3') one may apply the WKB solutions (16, 16') only far away from points $\varphi = 0$ and $\varphi = 2\pi$. In the vicinity of $\varphi = 0$ eq. (3'), (20) is solvable in terms of the parabolic cylinder functions. Matching the asymptotic behavior of the latter to the WKB-form (16, 16') makes it possible to find relations between coefficients (a, b) in (16) to the right of $\varphi = 0$, and the similar set (a', b'), to the left of it:

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \beta^* & \alpha^* \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \;\; ; \quad \left(|\alpha|^2 - |\beta|^2 = 1 \right) \quad (22)$$

Moving along φ from $\varphi = (0)_+$ and using (16), one reaches close to the point $\varphi = (2\pi)_-$. With the help of (22), the solution then may be transformed into the solution at $\varphi = (2\pi)_+$.

The periodicity (see (9)) leads to the equation:

$$R(l) \equiv |\alpha| \left(e^{+i\tilde{S}} + e^{-i\tilde{S}} \right) = 2\cos 2\pi\kappa \quad (23)$$

Here and below the dimensionless energy, l, is

$$l = 2(-\varepsilon)/\omega_c a^{1/2} \quad (24)$$

In (23) we used notations:

$$\alpha = |\alpha| \exp(i\theta) \;\; ; \;\; \tilde{S} = S(2\pi, l) - \theta(l) \quad (25)$$

($S(2\pi, l)$ coincides with (19) re-written in terms of l from (24)). Of the two solutions in eq. (23) we have to take

$$e^{i\tilde{S}} = \frac{\cos 2\pi\kappa}{|\alpha|} + \sqrt{\left(\frac{\cos 2\pi\kappa}{|\alpha|} \right)^2 - 1} \equiv \rho \quad (26)$$

Indeed, at $l \to +\infty$, $\alpha \Rightarrow 1$, eq. (25) for energy levels goes over into the WKB-result (18).

Introduce the function

$$\Phi(-\varepsilon) \equiv \Phi(l) = S(2\pi, l) - \theta(l) - \frac{1}{i} \ln \rho(l) \quad (27)$$

As we just mentioned, at large l solutions of $\Phi(l_n) = 2\pi n$ go over into the WKB-spectrum which at very large energies, $l \gg 1$, transforms into the spectrum of free electrons in the magnetic field. $\Phi(l)$ is a "monotonous" real function at $l \sim 1$. At $l < 0$ eq. (13) also determines the WKB-localized levels immediately below h_{max}. Therefore we may use (27) as the definition of $\lambda(E)$ (12), *including* energies in the vicinity of h_{max} where our phenomenon takes place. Correspondingly, the integral in (11) over λ acquires the meaning of the integration over energy: $d\lambda \Rightarrow (d\lambda/dE)dE \equiv (d\lambda/dl)\, dl$.

Consider (27) in some more details. The transition coefficient, $|\alpha|$, on the real axis is of the form:

$$|\alpha| = (1 + e^{-\pi l})^{1/2} \quad (28)$$

It has the branch-cuts at points $l_m = \pm(2m+1)i$ ($m = 0, 1, \ldots$) which should be chosen along *imaginary* axis, for $|\alpha|$ to be the single valued analytical function inside a strip near the real axis. The phase, $\theta(l)$, displays similar properties at $|l| \sim 1$ (see $\tilde{S}(l)$ (31)). As for $\rho(l)$ (26), the square root in (26) has singularities at:

$$l'_m = \pm(2m+1)i - \frac{1}{\pi} \ln(\sin^2 2\pi\kappa) \quad (29)$$

Bending the contour of integration over l into the upper half-plane, the main contribution into M_{osc} (11) is to be due to "nearest" singularities, at l_m and l'_m.

These values have the scale $|\lambda| \sim 1$, which, being re-written in terms of the energy, (24), results in:

$$|\varepsilon| \sim \omega_c a^{1/2} \sim h(\xi_0/d)^{1/2} \ll h \quad (30)$$

Eq. (30) justifies our implicit assumptions above that the effect is caused by the main peak, h_{max}. Other peaks, although being of the same scale, $h \sim v_F/d$, do not contribute to the effect produced by the fine structure (30) of the levels developing in the very vicinity of h_{max}.

The subsequent analysis although simple and straightforward, involves somewhat lengthy mathematical details to be published elsewhere. Below we provide only a sketch of a few major steps, before discussing the result.

Exact $\Phi(l)$, eq. (27), has no non-analytic $l\ln l$-term in (19) at small l. At $l\sim 1$ one gets ($c\sim 1$):

$$\tilde{S}(l)=S(2\pi,0)+\frac{1}{2}l\left[\ln\left(\sqrt{\frac{d}{\xi_0}}\right)+c\right]- \\ -\frac{1}{2i}\ln\left[\frac{\Gamma(\frac{1}{2}+\frac{il}{2})}{\Gamma(\frac{1}{2}-\frac{il}{2})}\right] \tag{31}$$

The important step is to calculate $\overline{|u_l|^2}$ in (11). In particular, the normalization is to be known with better accuracy than provided by the WKB-expressions of eq. (16, 16'). This can be achieved in frameworks of the above method. Fortunately, however, properties of the Bloch functions for a one-dimensional periodical potential are well studied. With the help of eq. (4.18) in ref. [8] and our eqs. (21) we derived:

$$\overline{|u_l|^2}=\frac{1}{2}-\pi a^{1/2}\sin 2\pi\kappa\left[\frac{dR}{dl}\right]^{-1} \tag{32}$$

where $R(l)$ is the R.H.S. of eq. (23). Expression for M_{osc} (11) is given as a sum of integrals, I_K, which may be re-written as integrals over the energy variable, l:

$$I_K=\int_{-\infty}^{+\infty}e^{iK\Phi(l)}\frac{d}{dl}\left(\overline{|u_l|^2}\right)dl \tag{33}$$

The constant, i.e. 1/2, having been eliminated in (33), the path of integration over real l can be transformed into two contours, C_1 and C_2, encircling the two branch-cuts caused by singular points of $\Phi(l)$, l_m, and l'_m of eq. (28, 29). With integrations over C_1 and C_2 now running parallel to the imaginary axis integrals become rapidly (exponentially) convergent, and integration back by parts is allowed. One obtains two integrals of the form:

$$I_K=\frac{iK\pi a^{1/2}\sin 2\pi\kappa}{2}\int\frac{\exp iK\Phi(l)dl}{(\sin^2 2\pi\kappa+e^{-\pi l})^{1/2}} \tag{34}$$

With $\Phi(l)$ given by (27) and (31), and the proper definition of the branches on the cuts, integrations can be completed analytically, particularly so in the limit $\ln\left(\sqrt{d/\xi_0}\right)\gg 1$. Without providing here the result explicitly, we use this latter assumption to discuss the order of magnitude for $I_{K=1}$:

$$I_{K=1}\propto a^{1/2}e^{iS(2\pi,0)}\left(\xi_0/d\right)^{1/4}\sim \\ \sim\Delta/\omega_c\left(\xi_o/d\right)^{7/4}e^{iS(2\pi,0)} \tag{35}$$

The numerical coefficient in (35) is large but there is a phase factor containing

$$S(2\pi,0)=\sqrt{2\Delta}/\omega_c\int_0^{2\pi}(h_{max}-h(\varphi))^{1/2}d\varphi \tag{36}$$

and the result has to be averaged yet over all trajectories.

Our estimates proceed in the following way. Since the Larmor orbit is so large ($r_L\gg d$), the maximum in $h(\varphi)$ must be practically the same for any trajectory, while its position, φ_0, varies. The position, φ_0, played no role in our analysis above. The integral in (36) is positive, therefore we write

$$e^{iS(2\pi,0)}=e^{i\overline{S(2\pi,0)}}e^{i\delta S}$$

where $\overline{S(2\pi,0)}$ is an average, i.e. *just a phase* (although unknown), while δS fluctuates around $\overline{S(2\pi,0)}$. Since $\overline{S(2\pi,0)}\sim\Delta/\omega_c\left(\xi_0/d\right)^{1/2}$ is large, $\langle e^{i\delta S}\rangle$ can be calculated using for δS the Gaussian distribution with $\overline{\delta S^2}=\overline{S(2\pi,0)}$. For M^S_{osc}/M^N_{osc} it gives:

$$\sim\Delta/\omega_c\left(\xi_0/d\right)^{7/4}\exp\left[-\Delta/\omega_c\left(\xi_0/d\right)^{1/2}\right] \tag{37}$$

The exponent (37) remains, of course, yet too small to expect that the dHvA effect may be measured experimentally in the regime (1). Nevertheless, we have demonstrated the effect of the local density of states characteristic of the superconducting state: an effective Dingle's temperature becomes reduced as $\Delta\Rightarrow\Delta(\xi_0/d)^{1/2}$ even for the isotropic BCS-like superconductor. In the anisotropic case such a reduction is expected to be stronger [9] $\Delta\Rightarrow\Delta(\xi_0/d)$, in the same regime (1).

To conclude, we demonstrated that, unlike normal metals where the dHvA-effect is caused by electron levels crossing the chemical potential, in the superconducting heterogeneous state oscillations are caused by levels crossing the energy threshold separating "localized" and itinerant states. We argue that this mechanism may be responsible for pronounced oscillations in the regime $B\sim H_{c2}$ where not only is the gap weaker, but is heterogeneous (periodic) itself, together with distributions of the field and currents.

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